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Measuring Mass & Force With A Balance

1. Scope

This paper addresses the measurement of mass and force (or pressure) with an electronic balance or scale [1]).

Any electronic balance that uses a physical principle measuring force falls within the scope of this text. Representative transducers of this class include those of the electrodynamic compensation principle [2]) and the strain gauge weighing cell.

On the contrary, mass comparison principles, such as the archetype equal beam balance, for example, do not belong to this class of force measuring devices, and hence, not all conclusions in this paper apply to them.

In this text the electrodynamic compensation principle is mainly used as a reference.

2. Introduction

The purpose of a balance is that it determines the mass of a body.

Most electronic balances do not measure mass directly; indeed, they measure force, namely the weight force of the object placed on the weighing platform. Nevertheless, the displayed result is expressed in a mass unit (kg, g, etc.). To justify this, the output of the internal force measurement transducer is scaled by a proportionality factor before the mass value is displayed. This scaling factor (calibration factor) is determined by a calibration weighing, i.e., by weighing a calibration weight of known mass.

Therefore, the following two corollaries hold for any balance (capable of performing a calibration, external or internal):

A) The object placed on the pan is subject to gravity acting at the location of the balance.
B) The purpose of calibrating a balance is to make the balance display the mass value of an object weighed.

Statement A) reflects the fact that the weight (force) G of the object is

$$G = mg \quad ,$$

where m is the object's mass, and g is the gravitational factor [3]) at the location of the balance.

From statement B) we conclude that

$$R = M$$

1) Typically, the term "balance" refers to high resolution low capacity laboratory weighing instruments, whereas the term "scale" refers to high(er) capacity weighing equipment with lower resolution. While this discussion applies equally to balances and scales, we use here the term "balance" for both.
2) also referred to as "electromagnetic force restoration", EMFR
3) $g \approx 9.77 \ldots 9.83$ N/kg, depending on the location, latitude, altitude, with these being the most important.

shall hold, where R is the displayed mass, or mass difference, and M is the value of the mass of the object weighed. For the time being, we will assume that the value M of the object's mass is identical to its physical mass m :

$$M = m \quad .$$

Whereas this identity seems rather obvious, it's not, as we will see later [4].

If these assumptions hold, the balance displays the mass of the object

$$R = M = m \quad ,$$

as desired.

3. Calibration of a Balance

To adjust the sensitivity of the balance, there has to be a scaling factor (calibration factor) along the signal processing chain, somewhere between the force transducer and the display. We state this as follows:

$$R = c\,G \quad ,$$

where c is this scaling or calibration factor. A successful calibration adjusts the calibration factor

$$c = \frac{R}{G}$$

such that

$$\Delta R := \Delta M$$

is met (statement B). Hence, the calibration factor can be deduced from

$$c = \frac{\Delta R}{\Delta G} = \frac{\Delta M}{\Delta m \cdot g} = \frac{1}{g}\left(\frac{\Delta M}{\Delta m}\right) \quad .$$

As previously discussed, we assume that the relation

$$\Delta M = \Delta m$$

holds for the time being. In this case, the calibration factor is given by

$$c = \frac{1}{g} \; .$$

In performing an actual calibration, the following three steps have to be carried out:

Calibration Procedure

i) Measure the mass of the empty platform (with or without some preload), representing a total mass of m_1 ;
the corresponding weight force is

$$G_1 = g_{CAL} m_1 \; ,$$

where g_{CAL} is the gravitational factor at the instant and location of calibration.

ii) Add the calibration mass m_{CAL} and measure again. We now have a total mass of

$$m_2 = m_1 + m_{CAL}$$

including the platform;
the corresponding weight force is

$$G_2 = g_{CAL} m_2 = g_{CAL}(m_1 + m_{CAL}) \quad .$$

iii) Computation of the calibration factor:
The load difference is

$$\Delta m = (m_1 + m_{CAL}) - m_1 = m_{CAL} \; ,$$

obviously, and the difference of the weight forces is

$$\Delta G = G_2 - G_1 = g_{CAL}(m_1 + m_{CAL}) - g_{CAL} m_1 = g_{CAL} m_{CAL} \; .$$

The expected difference in the displayed result is

$$\Delta R = M_{CAL} \; .$$

[4]) The concept of conventional mass purposely alters this relation.

We therefore get for the scaling factor

$$c = \frac{\Delta R}{\Delta G} = \frac{M_{CAL}}{m_{CAL}\, g_{CAL}} = \frac{1}{g_{CAL}}\left(\frac{M_{CAL}}{m_{CAL}}\right) \quad .$$

Still asking for identity between actual mass m and indicated mass M, namely

$$M_{CAL} = m_{CAL} \; ,$$

we eventually get for the calibration factor

$$c = \frac{1}{g_{CAL}} \quad .$$

This factor is stored in the internal memory of the balance — replacing the previous one — and residing there until the next successful calibration takes place. This factor, and the corresponding internal calibration mass values, are generally unavailable to the user. However, the net effect of calibrating the balance is transparent to the user, such that statement B) is fulfilled. This can be shown by substituting the calibration factor back into the signal path

$$R = cG = \left(\frac{1}{g_{CAL}}\right)(mg) = \left(\frac{g}{g_{CAL}}\right) m \quad .$$

If the same gravity acts on the mass as during the instant of calibration, i.e.,

$$g = g_{CAL} \; ,$$

the result displayed equals the mass placed on the weighing platform

$$R = \left(\frac{g_{CAL}}{g_{CAL}}\right) m = m \quad ,$$

as intended. However, moving the balance to a different place, the object weighed is generally subjected to a different gravitation. As can be deduced from the daily tides, gravitation changes even with time. Ultra high resolving balances are at the threshold to sense the tides [5]).

The adjustment deviation of the calibration mass limits the accuracy of the calibration and thus the accuracy of the balance's sensitivity. This deviation can be derived from the specification of the external or internal calibration mass.

Also, if the calibration mass does not comprise the full weighing range of the balance, there may occur an additional deviation stemming from the fact that a potential linearity deviation present at the value of the actual calibration mass is "leveraged" by deriving the overall sensitivity for the full weighing range. If the ultimate in sensitivity accuracy is required, an external calibration mass equal to the full range, or alternatively, to the specific working range, is recommended.

4. Buoyancy Effects

As long as a weighing is carried out in air, buoyancy will influence the result; a fact we have neglected so far.

Buoyancy acting on masses

According to Archimedes' law of buoyancy, an object's weight loss is equal to the weight of the fluid it displaces

$$G = mg - m_F\, g = (m - m_F)\, g \quad ,$$

m_F being the mass of the fluid displaced. If this fluid is air, its mass can be calculated from

$$m_F = Va = \left(\frac{m}{\rho}\right) a \quad ,$$

where V is the volume of the object weighed, ρ its density, and a the density of the air displaced [6]). We get

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5) The graviation's daily fluctuation amounts to about 0.2 … 0.3ppm of the nominal value.

6) R.S. Davis: Equation for the Determination of the Density of Moist Air (1981/91). Metrologia 1992, Vol. 29, pp. 67-70.
P. Giacomo: Equation for the Determination of the Density of Moist Air (1981). Metrologia 1982, Vol. 18, pp. 33-40.

$$G = \left[m - \left(\frac{m}{\rho}\right) a\right] \; g = m\left(1 - \frac{a}{\rho}\right) \; g = m_A g \quad .$$

Due to buoyancy it appears that there were less mass. The value m_A is therefore referred to as "apparent mass".

Clearly, the calibration mass is subject to buoyancy, too. Consequentially, a "real world calibration", i.e., one that takes buoyancy into account, produces a slightly different calibration factor

$$c = \frac{\Delta R}{\Delta G} = \frac{1}{g_{CAL}} \frac{M_{CAL}}{m_{CAL}\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} = \frac{1}{g_{CAL}} \left(\frac{M_{CAL}}{m_{CAL}}\right) \frac{1}{\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} \quad .$$

Now, using this calibration factor while performing a "real world weighing", i.e., one that takes buoyancy into account, this buoyancy effect should be canceled in the net result, since the weighed object is subject to buoyancy, too. We have as weight force of the object (considering buoyancy)

$$G = m\left(1 - \frac{a}{\rho}\right) \; g$$

and the signal processing with the calibration factor (considering buoyancy) gives

$$R = \frac{1}{g_{CAL}} \left(\frac{M_{CAL}}{m_{CAL}}\right) \frac{1}{\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} G \quad .$$

Hence we have

$$R = \frac{1}{g_{CAL}} \left(\frac{M_{CAL}}{m_{CAL}}\right) \frac{1}{\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} m\left(1 - \frac{a}{\rho}\right) g = \frac{g}{g_{CAL}} \left(\frac{M_{CAL}}{m_{CAL}}\right) \frac{\left(1 - \frac{a}{\rho}\right)}{\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} m \quad ;$$

still assuming that

$$M_{CAL} = m_{CAL} \text{ and } g = g_{CAL}$$

we finally get

$$R = \frac{\left(1 - \frac{a}{\rho}\right)}{\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} m \quad .$$

Provided the object weighed has the same density as the calibration mass

$$\rho = \rho_{CAL} \; ,$$

and the air density at the instant of weighing is equal to the air density at the instant of calibration

$$a = a_{CAL} \; ,$$

then we get, once more

$$R = m \quad .$$

Buoyancy Acting On the Balance

Buoyancy also affects the balance as a measuring instrument. All mechanical parts constituting the balance are subject to buoyancy, since they are submerged in air. Objects mechanically connected to the "input port" of the balance are affected, such as the weighing pan, the hanger, the lever (partially), if present—to mention a few. This leads to a zero shift of the displayed result. However, as soon as the balance is tared, or the difference of two readings is taken, this effect cancels. There is no path known through which buoyancy affects the sensitivity of the transducer.

One exception is the calibration mass, and hence the calibration process, determining the sensitivity of the balance, as discussed in the previous paragraph. However, when the calibration process is concluded, the sensitivity remains unaffected by buoyancy again [7]).

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[7]) Another exception is the influence of buoyancy during the manufacturing adjustment of the temperature compensation algorithm. To adjust assembled balances, they are subjected to various ambient temperatures, and at the same time they are loaded with test weights. As a matter of fact, for those balance series that do not have built in weights, that is, for those which need external weights heavier than about 1/2kg, brass is used as a raw material for these weights. Their density of approximately 8450kg/m3 is different from the OIML

5. The Concept of Conventional Mass

So far we have assumed that the value assigned to a mass and displayed when placed on the weighing pan is identical to its actual mass, namely

$$M_{CAL} = m_{CAL} \ .$$

However, to overcome the influence of air buoyancy when disseminating weights (weight standards), OIML [8]) has worked out a convention that is agreed upon by regulatory bodies, such that the actual mass of weights is adjusted to compensate for the apparent loss of mass in the presence of buoyancy. In short, the convention cancels the identity between assigned and actual mass, replacing it with the following relation between the two:

$$M = \frac{1-\frac{a_K}{\rho}}{1-\frac{a_K}{\rho_K}} m \quad \text{Definition of conventional mass } ^{9)},$$

where m is the actual mass of the weight, M is the conventional mass assigned to the weight, ρ is the actual density of the weight, ρ_K is the conventional density used for weight standards, and a_K is the conventional density assigned to air.

These conventional values are, by definition:

$\rho_K = 8000 \text{ kg/m}^3$ for the density of weight standards, and

$a_K = 1.2 \text{ kg/m}^3$ for the density of air.

This relation is derived from the convention, whose main idea is to forgo buoyancy correction in those cases, where a body has a density of 8000kg/m^3 and the mass comparison (i.e., comparison weighing) is carried out in air of density 1.2kg/m^3, and to correct by adding or removing extra mass for the manifesting buoyancy difference in all other cases, where the body's density is different from the conventional one.

Hence, no correction is made for buoyancy when a body possesses the conventional density (one of roughly heavy metals) and is weighed in air of conventional density (one of normal atmosphere).

The actual mass of a weight, given its conventional mass, can be determined by solving the definition formula for m:

$$m = \frac{1-\frac{a_K}{\rho_K}}{1-\frac{a_K}{\rho}} M \ .$$

Using this relationship for the calibration mass, we have to abandon the assumption we made until now, namely

$$M_{CAL} = m_{CAL} \ ,$$

and replace it with the relation of the conventional value

$$M_{CAL} = \frac{1-\frac{a_K}{\rho_{CAL}}}{1-\frac{a_K}{\rho_K}} m_{CAL} \ .$$

Substituting this relation into the original formula for the weighing result

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conventional density of 8000kg/m3 (see chapter 5). Therefore, a small buoyancy correction intrinsically gets caught in the temperature adjustment and compensation, because the density of the ambient air changes with temperature.

8) Organisation Internationale de Métrologie Légale

9) Valeur Conventionelle du Résultat des Pesées dans l'Air.
OIML Recommendation Internationale N° 33, Première édition 1973.
Conventional Value Of The Result Of Weighing In Air.
International Organization of Legal Metrology, International Recommendation No. 33, English Translation, 1979.

$$R = \frac{1}{g_{CAL}} \left(\frac{M_{CAL}}{m_{CAL}} \right) \frac{1}{\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} G \ ,$$

we get

$$R = \frac{1}{g_{CAL}} \frac{1 - \frac{a_K}{\rho_{CAL}}}{1 - \frac{a_K}{\rho_K}} \frac{1}{\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} G = \frac{1}{g_{CAL}} \frac{1 - \frac{a_K}{\rho_{CAL}}}{\left(1 - \frac{a_K}{\rho_K}\right)\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} G \ .$$

Because the weight force of the mass placed on the pan is still

$$G = m\left(1 - \frac{a}{\rho}\right) g \ ,$$

we eventually read from the balance

$$R = \frac{1}{g_{CAL}} \frac{1 - \frac{a_K}{\rho_{CAL}}}{\left(1 - \frac{a_K}{\rho_K}\right)\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} m \left(1 - \frac{a}{\rho}\right) g = \frac{g}{g_{CAL}} \frac{\left(1 - \frac{a_K}{\rho_{CAL}}\right)\left(1 - \frac{a}{\rho}\right)}{\left(1 - \frac{a_K}{\rho_K}\right)\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} m \ .$$

Again, if
the density of the weighed object is the same as the density of the calibration mass

$$\rho = \rho_{CAL} \ ,$$

the air density at the instant of weighing is equal to the air density at the instant of calibration

$a = a_{CAL}$, and

the gravity at the instant of weighing the mass is equal to the gravity at the instant of calibration

$$g = g_{CAL}$$

we get

$$R = \frac{\left(1 - \frac{a_K}{\rho_{CAL}}\right)}{\left(1 - \frac{a_K}{\rho_K}\right)} m \ .$$

This expression corresponds to the definition of conventional mass: R equals the conventional value M of the actual mass m weighed. Indeed, the balance <u>displays the conventional value</u> of the mass being weighed.

As we already have seen, if a weight conforms to the laws of conventional mass, its (actual) mass is generally different from the conventional (assigned) mass. This difference is defined by

$$\delta m := m - M$$

and its relative value amounts to

$$\frac{\delta m}{M} := \frac{m - M}{M} = \frac{m}{M} - 1 = \frac{1 - \frac{a_K}{\rho_K}}{1 - \frac{a_K}{\rho}} - 1 \approx a_K \left(\frac{1}{\rho} - \frac{1}{\rho_K} \right) \ .$$

This is the (relative) amount of mass required to compensate for extra buoyancy with respect to a weight standard of conventional density.

6. Measuring Force with a Balance

Substituting the weight force G by a force F from a different source — particularly not a weight force — acting on the balance, we have

$$R = c\,F$$

As a first approach we don't consider buoyancy, let alone the concept of conventional mass. We therefore apply the relation for the calibration factor

$$c = \frac{\Delta R}{\Delta G} = \frac{1}{g_{CAL}}\left(\frac{M_{CAL}}{m_{CAL}}\right) \quad ,$$

and the calibration mass

$$M_{CAL} = m_{CAL} \,,$$

and get

$$R = \frac{1}{g_{CAL}}\left(\frac{M_{CAL}}{m_{CAL}}\right) F = \frac{1}{g_{CAL}}\, F \quad .$$

Hence, the force acting on the balance can be derived from the balance's reading as follows

$$F = g_{CAL} R \ .$$

This is what we would expect on first sight — maybe with the exception, that the gravity at the instant of calibration must be used instead of the gravity acting at the instant of measuring the force.

Again, in the real world buoyancy is present, and using a balance, we have to deal with the convention of mass. Although this does not affect the source of force to be measured [10]), it does affect the calibration of the balance, and hence the displayed value, in to ways:

i) the calibration mass was subject to buoyancy at the time of calibration, and

ii) the calibration mass was adjusted according to the laws of conventional mass.

Therefore we repeat the deduction again, now considering the above complications. This goes as follows:

$$R = \frac{1}{g_{CAL}}\left(\frac{M_{CAL}}{m_{CAL}}\right)\frac{1}{\left(1-\frac{a_{CAL}}{\rho_{CAL}}\right)} G \quad \text{(calibration)},$$

$$M_{CAL} = \frac{1-\frac{a_K}{\rho_{CAL}}}{1-\frac{a_K}{\rho_K}}\, m_{CAL} \quad \text{(conventional mass of calibration weight)}.$$

With these relations, we get for the balance's reading

$$R = \frac{1}{g_{CAL}}\,\frac{\left(1-\frac{a_K}{\rho_{CAL}}\right)}{\left(1-\frac{a_K}{\rho_K}\right)}\,\frac{1}{\left(1-\frac{a_{CAL}}{\rho_{CAL}}\right)}\, F \ ,$$

Note that the actual air density does not show up in this formula, as there is no instantaneous influence; however, air density had its influence during calibration. We conclude that while calibration is affected, force measurements are not.

Given the balance's reading, we can solve for the force acting on the balance

$$F = \frac{\left(1-\frac{a_K}{\rho_K}\right)\left(1-\frac{a_{CAL}}{\rho_{CAL}}\right)}{\left(1-\frac{a_K}{\rho_{CAL}}\right)}\, g_{CAL} R \ .$$

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[10]) If it does, this effect has to be dealt with separately.

7. Measuring Pressure

In case F emerges from other sources, such as from a pressure p acting on a surface A, we get the force

$$F = pA \, .$$

We substitute this expression into the relation for the force loading of the balance, and we get

$$R = \frac{1}{g_{CAL}} \frac{\left(1 - \frac{a_K}{\rho_{CAL}}\right)}{\left(1 - \frac{a_K}{\rho_K}\right)} \frac{1}{\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)} pA \, .$$

Hence, from the balance's reading we can solve for the pressure

$$p = \frac{\left(1 - \frac{a_K}{\rho_K}\right)\left(1 - \frac{a_{CAL}}{\rho_{CAL}}\right)}{\left(1 - \frac{a_K}{\rho_{CAL}}\right)} \frac{g_{CAL} R}{A} \, .$$

8. Conclusion

- The calibration of a balance is meant to sustain the feature of the balance to display the mass value of the object placed on the balance.
- The calibration weight's mass is adjusted according to the rules defined by the OIML convention #33 of conventional mass; the purpose of this convention is to reduce buoyancy corrections, or to make them obsolete altogether, when comparing (calibration) weights.
- The reading of a balance, when loaded with a mass m is

$$R = \frac{g}{g_{CAL}} \frac{\left(1-\frac{a_K}{\rho_{CAL}}\right)\left(1-\frac{a}{\rho}\right)}{\left(1-\frac{a_K}{\rho_K}\right)\left(1-\frac{a_{CAL}}{\rho_{CAL}}\right)} m \ .$$

- The reading of a balance, when loaded with a force F is

$$R = \frac{1}{g_{CAL}} \frac{\left(1-\frac{a_K}{\rho_{CAL}}\right)}{\left(1-\frac{a_K}{\rho_K}\right)} \frac{1}{\left(1-\frac{a_{CAL}}{\rho_{CAL}}\right)} F \ .$$

- The reading of a balance, when loaded with a pressure p acting on a surface area A is

$$R = \frac{1}{g_{CAL}} \frac{\left(1-\frac{a_K}{\rho_{CAL}}\right)}{\left(1-\frac{a_K}{\rho_K}\right)} \frac{1}{\left(1-\frac{a_{CAL}}{\rho_{CAL}}\right)} pA \ .$$

Explanation of Symbols Used

R	is the balance's reading, in [kg]
m	is the mass being weighed on the balance, in [kg]
g	is the gravity at the instant of the mass being weighed, in [N/kg], nominally 9.81N/kg [11)]
ρ	is the density of the mass being weighed, in [kg/m^3]
a	is the air density at the instant of the mass being weighed, in [kg/m^3]
F	is the force acting on the balance, in [N]
g_{CAL}	is the gravity at the instant of calibration, in [N/kg]
ρ_{CAL}	is the actual density of the calibration mass, in [kg/m^3]
a_{CAL}	is the air density at the instant of calibration, in [kg/m^3]
ρ_K	is the conventional density for weights, 8000kg/m^3
a_K	is the conventional density for air, 1.2kg/m^3

[11)] also known as "gravitational acceleration", used with the unit [m/s^2]

9. Some Calibration Weights Data Of Mettler-Toledo Balances

Analytical Balances, manufactured before 1974:

Material:	Chrome-Nickel Steel
Designation:	—
Actual Density:	$(\rho_{CAL})_{nom} = 7770\,\mathrm{kg/m^3}$
Conventional Density:	$\rho_K = 8400\,\mathrm{kg/m^3}$ [12]
Relative Mass Difference [13]:	$\frac{\delta m}{M} = 11.6\times10^{-6}$
Magnetic Permeability:	—

Analytical Balances, including AE-, AT-, MT/UMT-Series manufactured since 1974, up to and including 1996:

Material:	Chrome-Nickel Steel
Designation:	X 4 CrNi 18 13 special (1.3941)
Density:	$(\rho_{CAL})_{nom} = 7960\,\mathrm{kg/m^3}$
Conventional Density:	$\rho_K = 8000\,\mathrm{kg/m^3}$
Relative Mass Difference:	$\frac{\delta m}{M} = 0.754\times10^{-6}$
Magnetic Permeability:	$(\mu_{rel})_{CAL} = 1.0033$

Analytical Balances: AT-, MT/UMT-Series, manufactured since 1997:

Material:	Chrome-Nickel Steel
Designation:	X 2 NiCrMoCu 25 20 5 (1.4539)
Density:	$(\rho_{CAL})_{nom} = 8006\,\mathrm{kg/m^3}$
Conventional Density:	$\rho_K = 8000\,\mathrm{kg/m^3}$
Relative Mass Difference:	$\frac{\delta m}{M} = -0.112\times10^{-6}$
Magnetic Permeability:	$(\mu_{rel})_{CAL} = 1.0038$

Precision Balances and some models of Analytical Balances, including AG-, PR-, PG-Series:

Material:	Chrome-Nickel Steel
Designation:	X 10 CrNi 18 9 (1.4305)
Density:	$(\rho_{CAL})_{nom} = 7900\,\mathrm{kg/m^3}$
Conventional Density:	$\rho_K = 8000\,\mathrm{kg/m^3}$
Relative Mass Difference:	$\frac{\delta m}{M} = 1.90\times10^{-6}$
Magnetic Permeability:	—

12) Before 1974, the value of conventional density was set to 8400kg/m^3.

13) Relative difference between actual mass and conventional mass: $\frac{\delta m}{M} = \frac{m_{CAL} - M_{CAL}}{M_{CAL}}$

where m_{CAL} actual mass, M_{CAL} conventional mass.